

УПРАЖНЕНИЕ ПО ДИФЕРЕНЦИАЛНИ УРАВНЕНИЯ I

1) Решете уравнението:

1.1. $(1 + y^2) dx - 4 \sin^2 x dy = 0$;

1.2. $\sqrt{1 - y^2} dx - 2 \cos^2 x dy = 0$;

1.3. $(1 - y^2) dx - 3x dy = 0$;

1.4. $\sqrt{1 + y^2} dx - x^3 dy = 0$;

1.5. $\sin^2 y dx + (1 + x^2) dy = 0$;

1.6. $\cos^2 y dx + \sqrt{1 - x^2} dy = 0$;

1.7. $\sqrt{1 + y^2} dx - (x^2 + 4) dy = 0$;

1.8. $(9 + y^2) dx - \sqrt{4 - x^2} dy = 0$;

1.9. $(1 + x^2) dx - 4 \sin y dy = 0$;

1.10. $(5x^4 + 2x) dx + 4 \cos y dy = 0$.

2) Решете уравнението:

2.1. $(9 + y^2) dx - 8xy dy = 0$;

2.2. $(1 + y^2) dx - 4x^3 y dy = 0$;

2.3. $(y^2 + 1) dx - y(x^2 + 1) dy = 0$;

2.4. $\sqrt{1 - y^2} dx - y\sqrt{1 - x^2} dy = 0$;

2.5. $x(y^2 + 1) dx + (x^2 + 1) dy = 0$;

2.6. $x\sqrt{y^2 + 1} dx + \sqrt{x^2 + 1} dy = 0$;

2.7. $x \sin^3 y dx - (x^2 + 4) \cos y dy = 0$;

2.8. $y \sin x dx + (y^2 + 4) \cos^2 x dy = 0$;

2.9. $e^x (y^2 + 1) dx - (e^x + 2) dy = 0$;

2.10. $e^y (x^2 + 1) dx - x(e^y + 4) dy = 0$.

3) Решете уравнението:

3.1. $y' = x^3 y$;

3.2. $y' = xy^4$;

3.3. $y' = \frac{x^4}{y^2}$;

3.4. $y' = \frac{y^3}{x}$;

3.5. $y' = \sqrt{\frac{x}{y}}$;

3.6. $y' = \sqrt{\frac{y}{x}}$;

3.7. $y' = y \sqrt[4]{x^3}$;

3.8. $y' = x \sqrt[5]{y^2}$;

3.9. $y' = e^{x-y}$;

3.10. $y' = e^{x+y}$.

4) Решете уравнението:

4.1. $e^y (1 + y') = 1$;

4.2. $(1 + x^2) yy' = 1 + y^2$;

4.3. $\sqrt{1 - x^2} yy' = y^2 - 4$;

4.4. $xy' = y \ln y$;

4.5. $xy' = \sin^2 y$;

4.6. $(4 + e^x) yy' = e^x$;

4.7. $y' = \sin x \cdot \cos^2 y$;

4.8. $y' = \cos x \cdot \sin^2 y$;

4.9. $y' = y \cdot \operatorname{tg} x$;

4.10. $y' = \sqrt{x} \cdot \operatorname{cot} g y$.

5) Решете задачата на Коши:

$$5.1. \begin{cases} (1+y^2) dx - 2xy dy = 0; \\ y(2) = 1 \end{cases};$$

$$5.2. \begin{cases} \sqrt{1-y^2} dx - 2xy dy = 0; \\ y(0) = e \end{cases};$$

$$5.3. \begin{cases} (1+y^2) dx - \cos^2 x dy = 0; \\ y(\pi) = 1 \end{cases};$$

$$5.4. \begin{cases} \sqrt{1-y^2} dx + \sin^2 x dy = 0 \\ y\left(\frac{\pi}{2}\right) = \frac{1}{2} \end{cases};$$

$$5.5. \begin{cases} e^x y dx - (e^x - 2) dy = 0; \\ y(0) = e \end{cases};$$

$$5.6. \begin{cases} (x^2 + x) dx - x \sin y dy = 0; \\ y(4) = 0 \end{cases};$$

$$5.7. \begin{cases} \sqrt{y+2} dx - \sqrt{1-x^2} dy = 0; \\ y(0) = 2 \end{cases};$$

$$5.8. \begin{cases} (1+y)^2 dx - (1+x^2) dy = 0; \\ y(0) = 1 \end{cases};$$

$$5.9. \begin{cases} xy dx - (x^2 + 4) dy = 0; \\ y(0) = 1 \end{cases};$$

$$5.10. \begin{cases} (1+2x^2) dx - x(y^2 + 1) dy = 0 \\ y(1) = 3 \end{cases}.$$

6) Решете задачата на Коши:

$$6.1. \begin{cases} y' \cdot \operatorname{tg} 2x = y \\ y\left(\frac{\pi}{4}\right) = e^2 \end{cases}; \quad 6.2. \begin{cases} y' \cdot \operatorname{tg} 3x = y \\ y\left(\frac{\pi}{6}\right) = e^3 \end{cases}; \quad 6.3. \begin{cases} y' \cdot \operatorname{cot} g 4x = y \\ y(0) = e^4 \end{cases}; \quad 6.4. \begin{cases} y' \cdot \operatorname{cot} g 5x = y \\ y(0) = e^5 \end{cases}$$

$$6.5. \begin{cases} y' = \frac{8y}{x^3}; \\ y(2) = e \end{cases}; \quad 6.6. \begin{cases} y' = \frac{y^4}{6x}; \\ y(e) = 1 \end{cases}; \quad 6.7. \begin{cases} y' = \frac{\sqrt{y}}{x}; \\ y(e) = 4 \end{cases}; \quad 6.8. \begin{cases} y' = \frac{y}{\sqrt{x}}; \\ y(9) = 1 \end{cases};$$

$$6.9. \begin{cases} y' = \cos 2x \cdot \sin^2 y \\ y(0) = \frac{\pi}{4} \end{cases}, \quad 6.10. \begin{cases} y' = \cos^2 x \cdot \sin 2y \\ y\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \end{cases}.$$

7) Решете уравнението:

$$7.1. x \cdot y' - 2y = x^2;$$

$$7.2. x \cdot y' - 3y = x^3;$$

$$7.3. x \cdot y' - 4y = x^4;$$

$$7.4. x \cdot y' - 5y = x^5;$$

$$7.5. y' - 3x^2 \cdot y = \frac{e^{-x^3}}{\cos^2 x};$$

$$7.6. y' + 2x \cdot y = \frac{e^{-x^2}}{\sin^2 x};$$

$$7.7. x \cdot y' - 3y = x^4 \cdot e^{5x};$$

$$7.8. x \cdot y' - 2y = x^3 \cdot \sin 4x;$$

$$7.9. y = x(y' - x \sin x);$$

$$7.10. y = x(y' - x e^x).$$

8) Решете уравнението:

$$8.1. y' = \frac{2x+3y}{x};$$

$$8.3. y' = \frac{y}{x} + e^{\frac{y}{x}};$$

$$8.5. y' = \frac{y}{x} + \cos^2\left(\frac{y}{x}\right);$$

$$8.7. xy' - y = xe^{\frac{2y}{x}};$$

$$8.9. xy' - y = x \cos^2\left(\frac{4y}{x}\right);$$

$$8.2. y' = \frac{2y-x}{x};$$

$$8.4. y' = \frac{y}{x} + \sin^2\left(\frac{y}{x}\right);$$

$$8.6. y' = \frac{y}{x} + \operatorname{tg}\left(\frac{y}{x}\right);$$

$$8.8. xy' - y = x \sin^2\left(\frac{3y}{x}\right);$$

$$8.10. y' = \frac{y}{x} \ln\left(\frac{y}{x}\right).$$

9) Решете уравнението::

$$9.1. y'' - 4y' + 5y = 0;$$

$$9.3. y'' - y' - 12y = 0;$$

$$9.5. y'' + 2y' + y = 0;$$

$$9.7. y'' + 4y' + 13y = 0;$$

$$9.9. y'' - 6y' + 10y = 0;$$

$$9.2. y'' + 2y' - 8y = 0;$$

$$9.4. y'' - 4y' + 4y = 0;$$

$$9.6. y'' - 10y' + 25y = 0;$$

$$9.8. y'' + 2y' + 10y = 0;$$

$$9.10. y'' - 2y' + 2y = 0.$$

10) Решете уравнението::

$$10.1. y'' - 4y' = 0;$$

$$10.3. y'' - 4y = 0;$$

$$10.5. y'' + 4y = 0;$$

$$10.7. y'' + y' = 0;$$

$$10.9. y'' + y = 0;$$

$$10.2. y'' - y = 0;$$

$$10.4. y'' - 9y' = 0;$$

$$10.6. y'' + 9y = 0;$$

$$10.8. y'' - 9y = 0;$$

$$10.10. y'' + 16y = 0.$$

11) Решете уравнението:

$$11.1. y''' - 4y'' - 12y' = 0;$$

$$11.3. y''' + 2y'' + 5y' = 0;$$

$$11.5. y''' + 6y'' + 10y' = 0;$$

$$11.7. y''' + y'' - 2y' = 0;$$

$$11.9. y''' - 6y'' + 9y' = 0;$$

$$11.2. y''' - 4y'' = 0;$$

$$11.4. y''' - y' = 0;$$

$$11.6. y''' + 25y' = 0;$$

$$11.8. y''' + 2y'' = 0;$$

$$11.10. y''' - 36y' = 0.$$

12) Решете уравнението:

$$12.1. y'' - 4y' + 4y = 20e^{-2x};$$

$$12.3. y'' - 4y' + 3y = 60e^{-2x};$$

$$12.5. y''' - 2y'' + 10y' = 40e^{-2x};$$

$$12.7. y''' - 6y'' = 30e^{-3x};$$

$$12.9. y'' + y' = 40e^{5x};$$

$$12.2. y''' - 6y'' + 9y' = 30e^{-3x};$$

$$12.4. y'' - 8y' + 16y = 80e^{-x};$$

$$12.6. y'' + 4y' + 5y = 30e^{-3x};$$

$$12.8. y''' - 4y' = 20e^{4x};$$

$$12.10. y'' - 8y' = 40e^{2x}.$$

13) Решете уравнението:

$$13.1. y'' - 4y' + 3y = 50e^x;$$

$$13.3. y'' - 4y' - 5y = 30e^{-x};$$

$$13.5. y'' - y' = 40e^x;$$

$$13.7. y'' + 2y' = 80e^{-2x};$$

$$13.9. y'' + 3y' + 2y = 50e^{-2x};$$

$$13.2. y'' + 2y' - 3y = 60e^x;$$

$$13.4. y'' + 6y' + 5y = 120e^{-x};$$

$$13.6. y'' - y = 90e^x;$$

$$13.8. y'' - 4y' = 60e^{2x};$$

$$13.10. y'' + 2y' - 8y = 80e^{2x}.$$

14) Решете уравнението:

$$14.1. y'' + 2y' + 17y = 34x + 64;$$

$$14.3. y'' + 4y' + 3y = 30x + 90;$$

$$14.5. y'' - 2y' + y = 6x;$$

$$14.7. y''' - y'' - 6y' = 36x + 24;$$

$$14.9. y''' + 8y'' + 16y' = 32x;$$

$$14.2. y''' + 4y'' + 13y' = 26x - 30;$$

$$14.4. y''' + y'' - 6y' = 24x;$$

$$14.6. y''' + y'' = 36x;$$

$$14.8. y''' - 2y'' = 12x + 30;$$

$$14.10. y''' + 5y'' = 30x.$$

15) Решете уравнението:

$$15.1. y''' - 2y'' + 10y' = 30x^2;$$

$$15.3. y''' + y'' - 6y' = 18x^2 + 36;$$

$$15.5. y''' + 2y'' = 24x^2 + 30x;$$

$$15.7. y''' - y'' = 12x^2 - 30;$$

$$15.9. y''' + 3y'' = 36x^2;$$

$$15.2. y'' - 4y' - 5y = 20x^2 + 10;$$

$$15.4. y'' - 4y' + 4y = 12x^2;$$

$$15.6. y''' - y' = 6x^2 + 4x + 10;$$

$$15.8. y''' + 4y' = 12x^2 + 8x + 15;$$

$$15.10. y'' + 2y' = 9x^2 + 18x.$$